# Numbering Systems

## What is a Numbering System?

A numbering system is a representation of a number line. The following are numbering system but there are more:

|  |  |
| --- | --- |
| Numbering System | Base |
| * Octal System | 8 (0, 1, 2, 3, 4, 5, 6, 7) |
| * Decimal System | 2 (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10) |
| * Hexadecimal System | 16 (0, 1, 2, ,3 ,4, 5, 6, 7, 8, 9, A, B, C, D, E, F ) |
| * Binary System | 2 (0, 1) |

Digits in any base always start at 0.

Note

## What is Positional Notation?

**Positional Notation** is a way of representing numbers, the position of each number has a place value; the digit is equal to its own number multiplied by its position value.

Example:

Consider the number 326632:

* Step 1: Separate the number 3, 2 and 6.
* Step 2: Multiple the positional value of the digit by the number. Example,3 x102
* Step 3: Add all the numbers together so it looks like

-> 3x105 + 2x104 + 6x103 + 6x102 + 2x101 + 3x100

## Addition in Binary

Addition of binary digits is very simpler to addition in base 10. E.g. 1 + 0 = 1, 1 + 1 = 0 (Carry 1) 0 + 0 = 0.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Carry | 1 |  | 1 | 1 | 1 | 1 | 1 |  |  |
| Num 1 |  | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| Num 2 |  | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
|  |  |  |  |  |  |  |  |  |  |
| Sum | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |

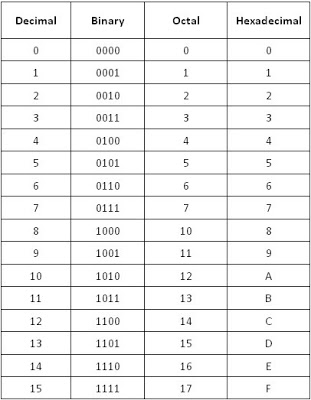
## Subtraction in Binary

Subtraction of binary digits is very simpler to addition in base 10. E.g. 0 – 0 = 0, 1 – 0 = 1, 1 – 1 = 0, 0 – 1 = 1 (Carry 1)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Carry |  |  |  |  |  |  |  |  |  |
| Num 1 |  | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| Num 2 |  | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
|  |  |  |  |  |  |  |  |  |  |
| Sum |  | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |

## Differences Between the Binary, Octal and Hexadecimal Number Systems

* Decimal is base 10 and has 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
* Binary is base 2 and has two digits (0 and 1)
* Octal is base 8 and has 8 digits (0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12)
* Hexadecimal is base 16 and has 16 digits / letters (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)



## Conversion from binary to decimal

Given a binary string: 1001011110  
We follow the steps in the diagram or as follows on how to convert it to decimal.

* Explanation: When a digit has a value 1, we carry it on to the next step, when it has a value 0, we do not.

We multiple the 1 by the number above the digit 1 and add that to the rest. This gives us our final answer in decimal.

1512 0256 0128 164 032 116 18 14 12 01

512 + 64 + 16 + 8 + 4 +2

= 606

## Conversion from decimal to binary

To convert a decimal number to binary, we need to divide the number by 2 and record the remainder. Once we divide it so much that the answer is 0, we stop; And we read the numbers from bottom to top (as shown by the arrow). The steps are shown in the diagram.

|  |  |
| --- | --- |
| Consider the number 2546 | Remainder |
| 2546 / 2 = 1273 | 0 |
| 1273 / 2 = 636 | 1 |
| 636 / 2 = 318 | 0 |
| 159 / 2 = 79 | 1 |
| 79 / 2 = 39 | 1 |
| 39 / 2 = 19 | 1 |
| 19 / 2 = 9 | 1 |
| 9 / 2 = 4 | 1 |
| 4 / 2 = 2 | 0 |
| 2 / 2 = 1 | 0 |
| 1 / 2 = 0 | 1 |

Read numbers from the bottom up.

Note

## Conversion from decimal to octal

To convert a decimal number to octal, we need to divide the number by 8 and record the remainder. Once we divide it to the point where the answer is 0, we stop; And we read the numbers from the bottom to top (as shown by the arrow). The steps are shown in the diagram.

|  |  |
| --- | --- |
| Consider the number 3584 | Remainder |
| 3584 / 8 = 448 | 0 |
| 448 / 8 = 56 | 0 |
| 56 / 8 = 7 | 0 |
| 7 / 8 = 0 | 7 |

Read numbers from the bottom up.

Note

## Conversion from octal to decimal

To convert octal to decimal start the decimal result at 0. Remove the most significant octal digit (leftmost) and add it to the result. If all octal digits have been removed, you’re done. Stop. Otherwise, multiply the result by 8. Repeat until all octal digits have been removed.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Octal Digit | Operation | Decimal Result | Operation | Decimal Result |
| 7767 | +7 | 7 | X8 | 56 |
| 767 | +7 | 63 | X8 | 508 |
| 67 | +6 | 514 | X8 | 4080 |
| 7 | +7 | 4087 | Done | 4087 |

## Conversion from decimal to hexadecimal

To convert a decimal number to hexadecimal, we need to divide the number by 16 and record the remainder. Once we divide it to the point where the answer is 0, we stop; And we read the numbers from the bottom to top (as shown by the arrow). The steps are shown in the diagram.

The alphabet in hexadecimal is 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

|  |  |
| --- | --- |
| Consider the number 8928 | Remainder |
| 8928 / 16 = 558 | 0 |
| 558 / 16 = 544 | 14 |
| 14 / 16 | 14 |

Read numbers from the bottom up.

Note

Therefore the answer is EE1.

## Conversion from hexadecimal to decimal

To convert a hexadecimal to decimal, Follow the steps in the Example: Convert AFB to decimal.

C x 163 = 12 x 4096 = 49151

A x 16² = 10 x 256 = 2560

+ F x 16¹ = 15 x 16 = 240

+ B x 16º = 11 x 1 = 11

= 51963 in base 10

Read numbers from the bottom up.

Note

## Conversion from binary to octal

To convert binary to octal; we group the binary digits in groups of three, starting from the right. Follow the steps in the example.

|  |  |  |
| --- | --- | --- |
| Step | Binary Digit | Octal Numbers |
| Step 1 | 10100110 | 010 100 110 |
| Step 2 | 10100110 | 28 48 68 |
| Step 3 | 10100110 | 2468 |

## Conversion from octal to binary

To convert binary to octal; Follow the steps in the example.

|  |  |  |
| --- | --- | --- |
| Step | Octal Number | Binary Digit |
| Step 1 | 77677765 | 78 78 68 78 78 78 68 58 |
| Step 2 | 77677765 | 1112 1112 1102 1112 1112 111 2 110 2 101 |
| Step 3 | 77677765 | 111111110111111111110101 |

## References

<https://encyclopedia2.thefreedictionary.com/digit+position> ( Positional Notation )

<https://bit.ly/2xZb7B2> slide 8 ( Differences between numbering systems )

<http://www.robotroom.com/NumberSystems4.html> ( Conversion of octal to decimal )

<http://www.math-only-math.com/binary-subtraction.html> ( Subtraction of Binary Digits)

<https://www.permadi.com/tutorial/numDecToHex/> ( Decimal to Hex )

<https://www.tutorialspoint.com/computer_fundamentals/computer_number_conversion.htm> (binary to octal & octal to binary )